Course Review

Chapter 4 - Exponential and Logarithmic Functions

4-1 One-to-one / Inverse functions

One-to-one functions: For every element in the range there is only element in the domain (horizontal line test)

Inverse functions: Exchange x and y values. Graph reflects across the y=x line. Domain and Range are swapped.

Finding an inverse function:
- Step 1: Replace f(x) with y
- Step 2: Interchange x and y (to get "implicit form")
- Step 3: Solve for y

Restricting domains to produce inverse functions:

Restrict domain of original function so that it become a one-to-one function

4-2 Exponential Functions: $f(x) = b^x$

Product Rule: $x^m \cdot x^n = x^{m+n}$ Quotient Rule: $\frac{x^m}{x^n} = x^{m-n}$ Power Rule: $(x^m)^n = x^{m \cdot n}$

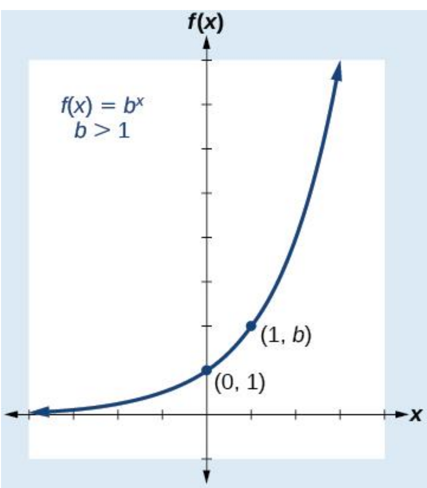
Distributive Rule for Products: $(x \cdot y)^m = x^m \cdot y^m$ Distributive Rule for Quotients: $(x/y)^m = \frac{x^m}{y^m}$

Zero Exponent: $x^0 = 1$ Negative Exponents: $x^{-m} = \frac{1}{x^m}$ Rational Exponents: $b^{(p/q)} = (q^{th} \text{ root of } b)^p$

Euler's Number: e = 2.718281828459045..... (irrational, transcendental)

Graphs of Exponential Functions:

Where b > 1

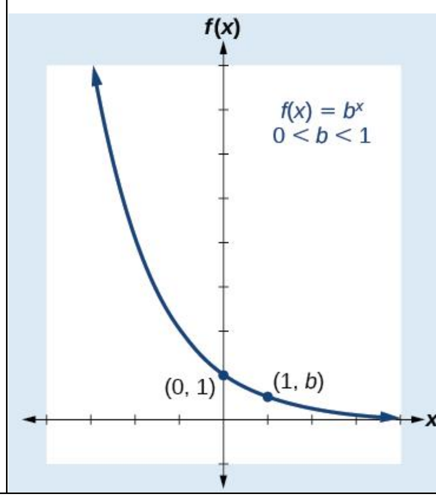


Domain: (-∞,∞)

Range: (0,∞)

Horizontal Asymptote: y = 0

Where 0 < b < 1



Domain: (-∞,∞)

Range: (0,∞)

Horizontal Asymptote: y = 0

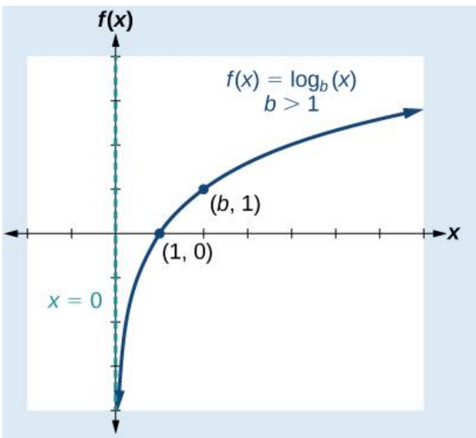
Exponential Equations - Same-base Method: If $b^p = b^q$, then $p = q$ (where $b \neq 1$)

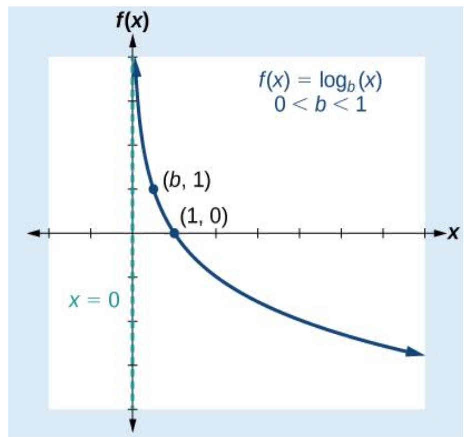
Exponential Equations - Graphing Method: Set both sides of equation = y, and find intersection of the 2 graphs

Exponential Equations - Logarithm Method: Take log of both sides. *(more information in section 4-3 below)*

4-3 Logarithmic Functions : $f(x) = \log_b(x)$

Graphs of Logarithmic Functions:

<table>
<tr>
<td>Where b > 1


Domain: $(0,\infty)$
Range: $(-\infty,\infty)$</td>
<td>Where 0 < b < 1


Domain: $(0,\infty)$
Range: $(-\infty,\infty)$</td>
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A logarithm IS an EXPONENT: $b^n = m$ ---> $\log_b(m) = n$ *(where* b *and* m *cannot be negative)*

A negative logarithm (n < 0) indicates division: $\log_b(m) = -n$ ---> $\frac{1}{b^m}$

Common Log: $\log_{10}(x)$ Natural Log: $\log_e(x)$ or $\ln(x)$

Change of base formula: $\log_b(x) = \frac{\log(x)}{\log(b)}$ *(can use any base, though base-10 and base-e are most useful)*

4-4 Properties of Logarithms

General: $\log_a(a) = 1$ $\log_a(1) = 0$ $\ln(e) = 1$ $\log_a(a)^x = x$ $\ln(e)^x = x$ $a^{\log a(x)} = x$ $e^{\ln(x)} = x$

Product Rule: $\log_b(x \cdot y) = \log_b (x) + \log_b (y)$ Division Rule: $\log_b(x/y) = \log_b(x) - \log_b(y)$

Power Rule: $\log_b(x^p) = p \cdot \log_b(x)$

4-5 Logarithmic and Exponential Equations

Solving exponential equations with different bases:

EX: $4^{(2x+2)} = 24$ --> $\log[4^{(2x+2)}] = \log[24]$ --> $(2x+2) \cdot \log(4) = \log[24]$ --> $2x+2 = \frac{\log(24)}{\log(4)}$ --> $2x+2 = 2.29$ --> $x = 0.145$

One-to-One Property of Logarithms: If $\log_b M - \log_b N$, then $M = N$

4-6 Compound Interest

Simple Interest: **$I = Prt$** *(where I = interest; P = principal; r = interest rate; t = time)*

Compound Interest: **$A = P(1 + \frac{r}{n})^{nt}$** *(where A = balance after "t" years; P = principal; r = interest rate; n = number of times per year that the interest is compounded; t = time)*

Continuous Compounding: **$A = Pe^{rt}$** *(where A = balance after "t" years; P = principal; r = interest rate; t = time)*

Effective Interest Rate: **$r = (1 + \frac{i}{n})^n - 1$** *(where r =effective interest rate; i = stated interest; n = number of times per year that the interest is compounded;)*

4-7 Growth and Decay

Exponential Growth: **$A(t) = A_0(1 + r)^t$** *(where $A(t)$ = amount after "t" time; A_0 = original amount; r = rate of increase; t = time)*

Exponential Decay: **$A(t) = A_0(1 - r)^t$** *(where $A(t)$ = amount after "t" time; A_0 = original amount; r = rate of increase; t = time)*

Radioactive/Continuous Growth: **$A(t) = A_0e^{kt}$** *(where $A(t)$ = amount after "t" time; A_0 = original amount; t = time; k = positive number related to growth rate)*

Radioactive/Continuous Decay: **$A(t) = A_0e^{-kt}$** *(where $A(t)$ = amount after "t" time; A_0 = original amount; t = time; k = negative number related to decay rate)*

Half Life: **$A(t) = A_0 \frac{1}{2}^{(t/n)}$** *(where $A(t)$ = amount after "t" time; A_0 = original amount; t = time; h = half life)*

4-8 Log, Exp, and Logistic Models

Logistical Growth Model *(An exponential model that can model situations where the growth of the dependant variable is limited):* **$P(t) = \frac{c}{1 + ae^{-bt}}$** $c > 0$ and $b > 0$

(where c = upper limit "carrying capacity"; a = constant; b = growth rate)