Course Review

Chapter 6 - Analytic Trigonometry

6-1 Inverse Sin, Cos, Tan *(input proportion, output angle)*

\[ \text{arcsin} = \sin^{-1}; \quad \sin^{-1}(y/r) = \theta \]

Restrictions: D: [-1,1]; R: [-\(\pi/2\), \(\pi/2\)]

\[ \text{arccos} = \cos^{-1}; \quad \cos^{-1}(x/r) = \theta \]

Restrictions: D: [-1,1]; R: [0, \(\pi\)]

\[ \text{arctan} = \tan^{-1}; \quad \tan^{-1}(y/x) = \theta \]

Restrictions: D: (-\(\infty\),\(\infty\)); R: [-\(\pi/2\), \(\pi/2\)]

6-2 Inverse Sec, Csc, Cot *(input proportion, output angle)*

\[ \text{arccsc} = \csc^{-1}; \quad \csc^{-1}(y/r) = \theta \]

Restrictions: D: (-\(\infty\), 1] \cup [1, \(\infty\)]; R: [-\(\pi/2\), 0) \cup (0, \(\pi/2\)]

\[ \text{arcsec} = \sec^{-1}; \quad \sec^{-1}(x/r) = \theta \]

Restrictions: D: (-\(\infty\),-1] \cup [1, \(\infty\)]; R: [0, \(\pi/2\)] \cup (\(\pi/2\), \(\pi\)]

\[ \text{arccot} = \cot^{-1}; \quad \cot^{-1}(y/x) = \theta \]

Restrictions: D: (-\(\infty\),\(\infty\)); R: [0, \(\pi\)]

6-3 Trigonometric Identities *(Magic Hexagon)*

Reciprocal Identities:

\[ \sin(x) = 1/\csc(x); \quad \cos(x) = 1/\sec(x); \quad \tan(x) = 1/\cot(x) \]

\[ \csc(x) = 1/\sin(x); \quad \sec(x) = 1/\cos(x); \quad \cot(x) = 1/\tan(x) \]

Quotient Identities:

\[ \tan(x) = \sin(x)/\cos(x); \quad \tan(x) = \sec(x)/\csc(x) \]
\[ \sin(x) = \cos(x)/\cot(x); \quad \sin(x) = \tan(x)/\sec(x) \]
\[ \cos(x) = \cot(x)/\csc(x); \quad \cos(x) = \sin(x)/\tan(x) \]
\[ \cot(x) = \csc(x)/\sec(x); \quad \cot(x) = \cos(x)/\sin(x) \]
\[ \csc(x) = \sec(x)/\tan(x); \quad \csc(x) = \cot(x)/\cos(x) \]
\[ \sec(x) = \tan(x)/\sin(x); \quad \sec(x) = \csc(x)/\cot(x) \]

Product Identities *(cross-multiply reciprocal and quotient identities)*

Complementary Angles

\[ \sin(x) = \cos(90^\circ-x); \quad \cos(x) = \sin(90^\circ - x) \]

\[ \tan(x) = \cot(90^\circ-x); \quad \cot(x) = \tan(90^\circ-x) \]
\[ \sec(x) = \csc(90^\circ-x); \quad \csc(x) = \sec(90^\circ-x) \]

Pythagorean Identities

\[ \sin^2\theta + \cos^2\theta = 1; \quad \tan^2\theta = 1 + \sec^2\theta; \quad 1 + \cot^2\theta = \csc^2\theta; \]

Even/Odd Trig Functions

\[ \text{ODD: } \sin(-x) = -\sin(x); \quad \csc(-x) = -\csc(x); \quad \tan(-x) = -\tan(x); \quad \cot(-x) = -\cot(x) \]

\[ \text{EVEN: } \cos(-x) = \cos(x); \quad \sec(-x) = \sec(x) \]

6-4 Sum and Difference Formulas

6-5 Double-Angle and Half-Angle Formulas

6-6 Product-to-Sum and Sum-to-Product
6-7 Trig Equations 1

How to solve trigonometric equations

Step 1: Look for any useful trig identities and substitute, if possible
Step 2: Optional - Substitute variables for trig functions (record variables and values!)
Step 3: Set equation equal to zero, if necessary
Step 4: Solve for variable by factoring, completing the square, or quadratic formula, as needed.
Step 5: If you substituted variables for trig functions (above), put trig functions back into equation
Step 6: Solve the function
Step 7: Reject any impossible solutions
Step 8: Remember that the solutions represent reference angles - Solve for all valid solutions based on sign and quadrant ("Save All The Children")

6-8 Trig Equations 2

How to solve trigonometric equations

Step 1: Look for any useful trig identities and substitute, if possible
Step 2: Optional - Substitute variables for trig functions (record variables and values!)
Step 3: Set equation equal to zero, if necessary
Step 4: Solve for variable by factoring, completing the square, or quadratic formula, as needed.
Step 5: If you substituted variables for trig functions (above), put trig functions back into equation
Step 6: Solve the function
Step 7: Reject any impossible solutions
Step 8: Remember that the solutions represent reference angles - Solve for all valid solutions based on sign and quadrant ("Save All The Children")